GEOMETRICAL OPTICS

1. Reflection of Light

(b) $\angle i = \angle r$

1.3 Characteristics of image due to Reflection by a Plane Mirror:

(a) Distance of object from mirror = Distance of image from the mirror.

(b) The line joining a point object and its image is normal to the reflecting surface.

(c) The size of the image is the same as that of the object.

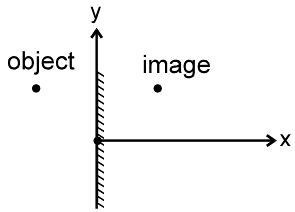
(d) For a real object the image is virtual and for a virtual object the image is real

2. Relation between velocity of object and image :

From mirror property : $x_{im} = - x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$

Here x_{im} means 'x' coordinate of image with respect to mirror.

Similarly others have meaning.



Differentiating w.r.t time , we get

$$V_{(im)x} = -V_{(om)x}; \qquad V_{(im)y} = V_{(om)y}; \qquad V_{(im)z} = V_{(om)z},$$

3. Spherical Mirror

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{R} = \frac{1}{f} \qquad ..... \qquad \text{Mirror formula}$$

x co–ordinate of centre of Curvature and focus of Concave mirror are negative and those for Convex mirror are positive. In case of mirrors since light rays reflect back in - X direction, therefore **-ve sign of v indicates real image and +ve sign of v indicates virtual image**

(b) Lateral magnification (or transverse magnification)

$$m = \frac{h_2}{h_1} \qquad\qquad m = -\frac{v}{u}.$$

(d) On differentiating (a) we get $\frac{dv}{du} = -\frac{v^2}{u^2}$.

(e) On differentiating (a) with respect to time we get

$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$,where $\frac{dv}{dt}$ is the velocity of image along Principal axis and $\frac{du}{dt}$ is the velocity of object along Principal axis. Negative sign implies that **the image , in case of mirror, always moves in the direction opposite to that of object.This discussion is for velocity with respect to mirror and along the x axis.**

(f) **Newton's Formula**: $XY = f^2$

X and Y are the distances (along the principal axis) of the object and image respectively from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

(g) Optical power of a mirror (in Diopters) = $\frac{1}{f}$

f = focal length with sign and in meters.

(h) If object lying along the principal axis is not of very small size, the longitudinal magnification = $\frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

4. Refraction of Light

vacuum. $\mu = \dfrac{\text{speed of light in vacuum}}{\text{speed of light in medium}} = \dfrac{c}{v}$.

4.1 Laws of Refraction (at any Refracting Surface)

(b) $\dfrac{\text{Sini}}{\text{Sinr}}$ **=** Constant for any pair of media and for light of a given wave length. This is known as Snell's Law. More precisely,

$$\frac{\text{Sin i}}{\text{Sin r}} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$$

4.2 Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at $\angle$i and refracted at $\angle$r is given by $\delta = |i - r|$.

5. Principle of Reversibility of Light Rays

A ray travelling along the path of the reflected ray is reflected along the path of the incident ray. A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus the incident and refracted rays are mutually reversible.

7. Apparent Depth and shift of Submerged Object

At near normal incidence (small angle of incidence i) apparent depth (d′) is given by:

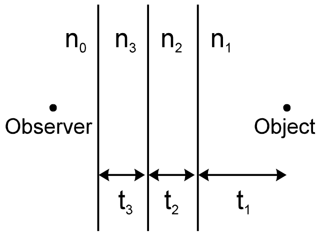
$$d' = \frac{d}{n_{relative}} \Rightarrow \quad n_{relative} = \frac{n_i (\text{R.I. of medium of incidence})}{n_r (\text{R.I. of medium of refraction})}$$

$$\text{Apparent shift} = d\left(1 - \frac{1}{n_{rel}}\right)$$

Refraction through a Composite Slab (or Refraction through a number of parallel media, as seen from a medium of R. I. n_0)

Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1\,rel}} + \frac{t_2}{n_{2\,rel}} + \frac{t_3}{n_{3\,rel}} + \ldots\ldots\ldots + \frac{t_n}{n_{n\,rel}}$$



$$\text{Apparent shift} = t_1\left[1-\frac{1}{n_{1rel}}\right] + t_2\left[1-\frac{1}{n_{2rel}}\right] + \ldots\ldots + \left[1-\frac{n}{n_{n\,rel}}\right]$$

8. Critical Angle and Total Internal Reflection (T. I. R.)

$$C = \sin^{-1}\frac{n_r}{n_d}$$

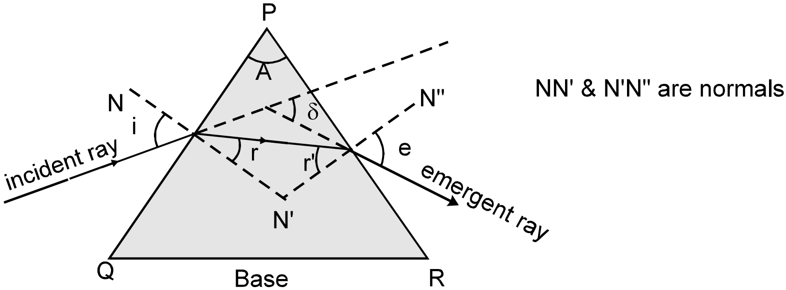
(i) Conditions of T. I. R.

(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle (i > c).

9. Refraction Through Prism

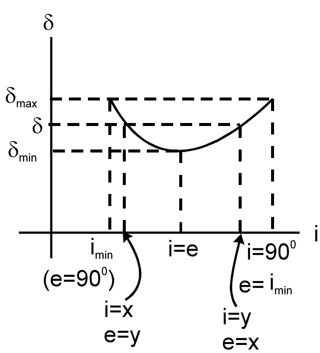
9.1 Characteristics of a prism



$$\delta = (i + e) - (r_1 + r_2) \text{ and } r_1 + r_2 = A$$

$$\therefore \quad \delta = i + e - A.$$

9.2 Variation of δ versus i



(1) There is one and only one angle of incidence for which the angle of deviation is minimum.

(2) When $\delta = \delta_{min}$, the angle of minimum deviation, then i = e and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that $\delta_{min} = 2i_{min} - A$ where i_{min} = angle of incidence for minimum deviation and r = A/2.

$$\therefore \; n_{rel} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \quad \text{where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n - 1)A$ (for small values of $\angle A$)

(3) For a thin prism ($A \le 10^o$) and for small value of i, all values of

$$\delta = (n_{rel} - 1)A \qquad \text{where } n_{rel} = \frac{n_{prism}}{n_{surrounding}}$$

10. Dispersion Of Light

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

The refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula $n(\lambda) = a + \frac{b}{\lambda^2}$ where a and b are positive constants of a medium.

Angle between the rays of the extreme colours in the refracted (dispersed) light is called **angle of dispersion.**

For prism of small 'A' and with small 'i' : $\theta = (n_v - n_r)A$

Deviation of beam(also called mean deviation) $\delta = \delta_y = (n_y - 1)A$

Dispersive power (ω) of the medium of the material of prism is given by:

$$\omega = \frac{n_v - n_r}{n_y - 1}$$

For small angled prism ($A \le 10^o$) with light incident at small angle i :

$$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y}$$

$$= \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

[$n_y = \dfrac{n_v + n_r}{2}$ if n_y is not given in the problem]

$\omega = \dfrac{\delta_v - \delta_r}{\delta_y} = \dfrac{n_v - n_r}{n_y - 1}$ [take $n_y = \dfrac{n_v + n_r}{2}$ if value of n_y is not given in the problem]

n_v, n_r and n_y are R. I. of material for violet, red and yellow colours respectively.

11. Combination of Two Prisms

Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.

(a) Direct Vision Combination (dispersion without deviation)

The condition for direct vision combination is :

$$\left[\frac{n_v + n_r}{2} - 1\right] A = \left[\frac{n'_v + n'_r}{2} - 1\right] A' \Leftrightarrow [n_y - 1]\, A = [n'_y - 1]\, A'$$

(b) Achromatic Combination (deviation without dispersion.)

Condition for achromatic combination is: $(n_v - n_r) A = (n'_v - n'_r) A'$

12. Refraction at Spherical Surfaces

For paraxial rays incident on a spherical surface separating two media:

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

where light moves from the medium of refractive index n_1 to the medium of refractive index n_2.

Transverse magnification (m) (of dimension perpendicular to principal axis)

due to refraction at spherical surface is given by $m = \dfrac{v - R}{u - R} = \left(\dfrac{v/n_2}{u/n_1}\right)$

13. Refraction at Spherical Thin Lens

A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends.

For a spherical, thin lens having the same medium on both sides:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \quad \text{where } n_{rel} = \frac{n_{lens}}{n_{medium}}$$

$$\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$ → Lens Maker's Formula

$$m = \frac{v}{u}$$

Combination Of Lenses: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} \ldots$

OPTICAL INSTRUMENT

SIMPLE MICROSCOPE

- Magnifying power : $\frac{D}{U_0}$
- when image is formed at infinity $M_\infty = \frac{D}{f}$
- When change is formed at near print D. $M_D = 1 + \frac{D}{f}$

COMPOUND MICROSCOPE

Magnifying power	Length of Microscope
$M = \frac{V_0 D_0}{U_0 U_e}$	$L = V_0 + U_e$
$M_\infty = \frac{V_0 D}{U_0 f_e}$	$L = V_0 + f_e$
$M_D = \frac{V_0}{U_0}\left(1 + \frac{D}{f_e}\right)$	$L_D = V_0 + \frac{D.f_e}{D + f_e}$

Astronomical Telescope

Magnifying power

$$M = \frac{f_0}{\mu_e}$$

$$M_\infty = \frac{f_0}{f_e}$$

$$M_D = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$$

Length of Microscope

$$L = f + u_e.$$

$$L = f_0 + f_e$$

$$L_D = f_0 + \frac{Df_e}{D+f_e}$$

Terrestrial Telescope

Magnifying power

$$M = \frac{f_0}{U_e}$$

$$M_\infty = \frac{f_0}{f_e}$$

$$M_D = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$$

Length of Microscope

$$L = f_0 + 4f + U_e.$$

$$L = f_0 + 4f + f_e.$$

$$L_D = f_0 + 4f + \frac{Df_e}{D+f_e}$$

Galilean Telescope

Magnifying power

$$M = \frac{f_0}{U_e}$$

$$M_\infty = \frac{f_0}{f_e}$$

$$M_D = \frac{f_0}{f_e}\left(1-\frac{f_e}{d}\right)$$

Length of Microscope

$$L = f_0 - U_e.$$

$$L = f_0 - f_e.$$

$$L_D = f_0 - \frac{f_e D}{D-f_e}$$

Resolving Power

Microscope $R = \frac{1}{\Delta d} = \frac{2\mu \sin\theta}{\lambda}$

Telescope. $R = \frac{1}{\Delta\theta} = \frac{a}{1.22\lambda}$